Reality Check: The Parameter Swap

The "Ethereal" vs. The "Real"

In Chapter 9 and Section 10.1, we operated in a world of **Statistical Perfection**. We assumed we knew the "Truth" (p or σ). In the *Clocktower* and the *Water Audit*, we had a manufacturer's guarantee or a census to lean on.

But as a Lead Engineer, you will quickly find that the "Truth" is usually hidden. We don't know the population noise (σ). We only have the noise we can see in our hands: the **Sample Standard Deviation** (s).

1. The Notation Upgrade

Task: Rewrite the Null and Alternative hypotheses by swapping the Chapter 9 parameter (p) for the Chapter 10 parameter (μ). Note that we use the subscript "0" to represent our Null value or "Status Quo."

Chapter 9 Reference (Proportions): $H_0 : p = p_0$ vs. $H_a : p \neq p_0$

2. The Standard Error as a "Proxy Ruler"

In the 10.1 Audit, we calculated the "Sniper Rifle" precision using $\sigma_{\bar{x}} = \sigma/\sqrt{n}$. Now, since σ is missing, we use s as a **proxy**. We view the **Standard Error** (SE) as our **best guess** of the true standard deviation of the sampling distribution ($\sigma_{\bar{x}}$):

$$SE = \frac{s}{\sqrt{n}}$$

Discussion Box: If your sample is tiny (like $n = 5$), why does our "ruler" feel more like a **rubber band** than a **steel rod**?

3. Summarizing the Sample

Using the `mean()` and `sd()` functions in R, summarize the following 5 Statypus Water bottles:

500.2, 500.5, 500.1, 500.8, 500.4

Calculated Sample Mean ($\bar{x}$):

Calculated Sample Standard Deviation (s):

Your "Best Guess" Ruler (SE):

4. The Lead Engineer's Audit

Translate these business scenarios into the new μ notation.

Scenario	**Your Hypotheses (H_0 and H_a)**
The label says 500mL. We want to prove we are over-filling.	$H_0 : \mu = 500 \quad H_a : \mu > 500$
The machine is set to 500.5mL. We suspect the calibration has shifted.	
A competitor claims their bottles average 501mL. We think they are lying (less).	

5. The Universal Logic

Task: Sketch a **Standard Normal Curve (z)**. Center your curve at 0 and mark the scale in standard deviations from -3 to 3. On this curve, mark where your calculated Standard Error (SE) falls relative to the center.

6. Final Audit Reflection

If you find that your sample mean ($\bar{x}$) is 3 Standard Errors away from the Null (μ_0), mark that spot on your curve in Part 5. Does the **logic** of the "Verdict" change just because the units changed? Is a 3-standard-deviation event any less rare just because we are auditing a mean instead of a proportion?