The Safety Net: Why the t-distribution?

1. The Penalty for Guessing

Because we are using s (a sample guess) instead of σ (the population truth), our ruler is inherently less stable. To account for this uncertainty, we swap the Normal (z) curve for the **t-distribution**.

The t-curve is indexed by **Degrees of Freedom (df)**. For a single sample mean, the formula is:

$$df = n - 1$$

Task: Calculate the df for the following Lead Engineer audits:

- You audit a single case ($n = 12$): df = __________
- You audit a double-case batch ($n = 24$): df = __________
- You audit a full pallet ($n = 144$): df = __________

2. Identifying the "Safety Net"

Open `visualizingt.r` in RStudio and run **Lines 8 through 34**. This plot compares the "Steel Rod" (Normal) vs. the "Rubber Band" (t, $df = 2$).

Visual Audit: Look at the tails of the two curves (the areas far to the left and right).

- Which curve stays higher off the x-axis in the tails? ____________________
- In your own words, why does the t-distribution need these "thicker" tails when we only have a tiny amount of data?

3. The Convergence (Single Case vs. Pallet)

As our sample size (n) increases, our estimate of the "noise" (s) becomes more stable. The "Rubber Band" begins to tighten until it is as rigid as the "Steel Rod."

Task: Run **Lines 37 through 58** in your script. This overlays three curves: the Normal curve, a t-curve for a single case ($df = 11$), and a t-curve for a full pallet ($df = 143$).

The Convergence Sketch

(Sketch the three curves below. Use a solid line for the Normal curve and dashed/dotted lines for the two t-curves. Label the "Safety Net" gap between them.)

The "Critical" Audit: In your R plot, notice where the curves cross the 5% threshold (the "tails").

- For the **Normal curve**, the "Rare Event" boundary is at ± 1.96.
- For the **Single Case ($df = 11$)**, the boundary is further out, at ± 2.20.

Question: Why does the t-distribution move the "goalposts" further out when n is small? If we kept the goalposts at 1.96 for a sample of only 12 bottles, would we be *more* likely or *less* likely to accidentally claim a "Rare Event" that was actually just random noise?

4. Final Reflection: The Law of Certainty

As our sample size (n) grows, our "Best Guess" (s) becomes so reliable that the t-distribution eventually turns back into the ________________________ distribution.

Question: Look at your sketch on the previous page. As n increases from 12 to 144, what happens to the "Safety Net" (the gap between the curves)? Why does having more data allow us to shrink this margin?

Question: Imagine you find a sample mean that is 3 standard errors away from the Null.

- With $n = 3$, the t-curve is so thick that 3 standard errors is still considered "common."
- With $n = 1000$, the curve is so thin that 3 standard errors is a "Slam Dunk."

Explain how the extra area in the t-distribution's tails protects you from claiming a discovery when you actually just have a shaky estimate.